Ensemble Kalman Filter: status and new ideas

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UMCP

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Ensemble Kalman Filter: status and new ideas

- EnKF and 4D-Var are in a friendly competition
- Jeff Whitaker results: EnKF better than GSI
- In Canada: 4D-Var & EnKF the same in the NH and EnKF is better in the SH
- EnKF needs no adjoint model, priors, it adapts to changes in obs, it can even estimate ob errors
- We take advantage of ideas and methods developed for 4D-Var and easily adapt them into the LETKF (Hunt et al., 2007)

Current comparisons:

Inter-comparison of 4D-Var and EnKF systems for operational deterministic NWP

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WWRP/THORPEX Workshop on 4D-VAR and Ensemble Kalman Filter Intercomparisons

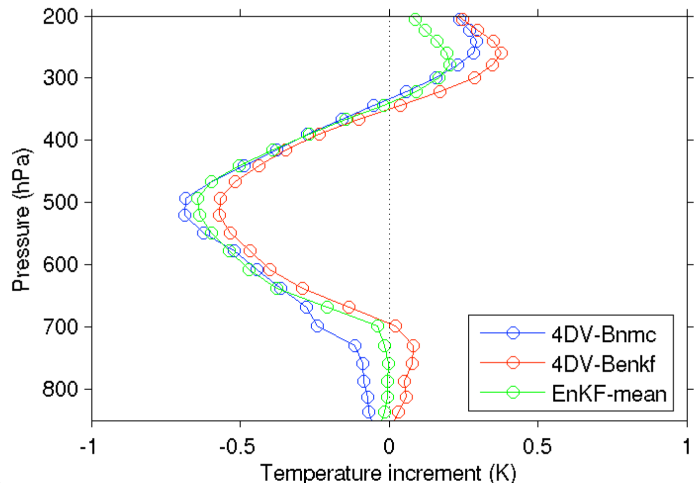
Buenos Aires, Argentina

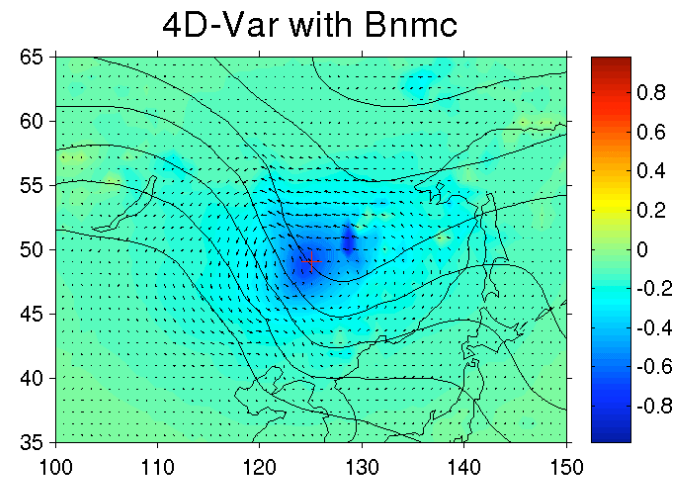
November 13, 2008

Single observation experiments

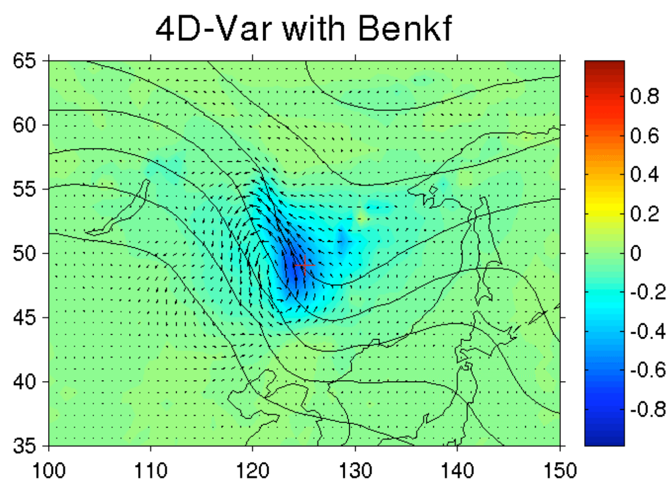
Difference in temporal covariance evolution

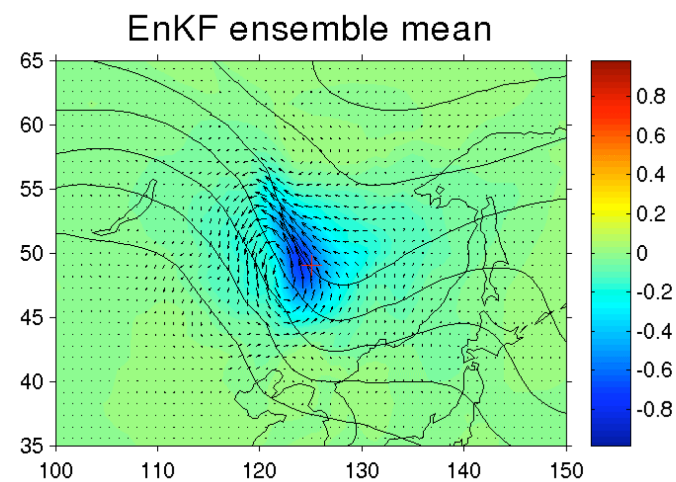
- radiosonde temperature observation at 500hPa
- observation at **middle of assimilation window (+0h)**
- with same B, increments very similar from **4D-Var**, **EnKF**
- contours are 500hPa GZ background state at 0h (ci=10m)





contour plots at 500 hPa



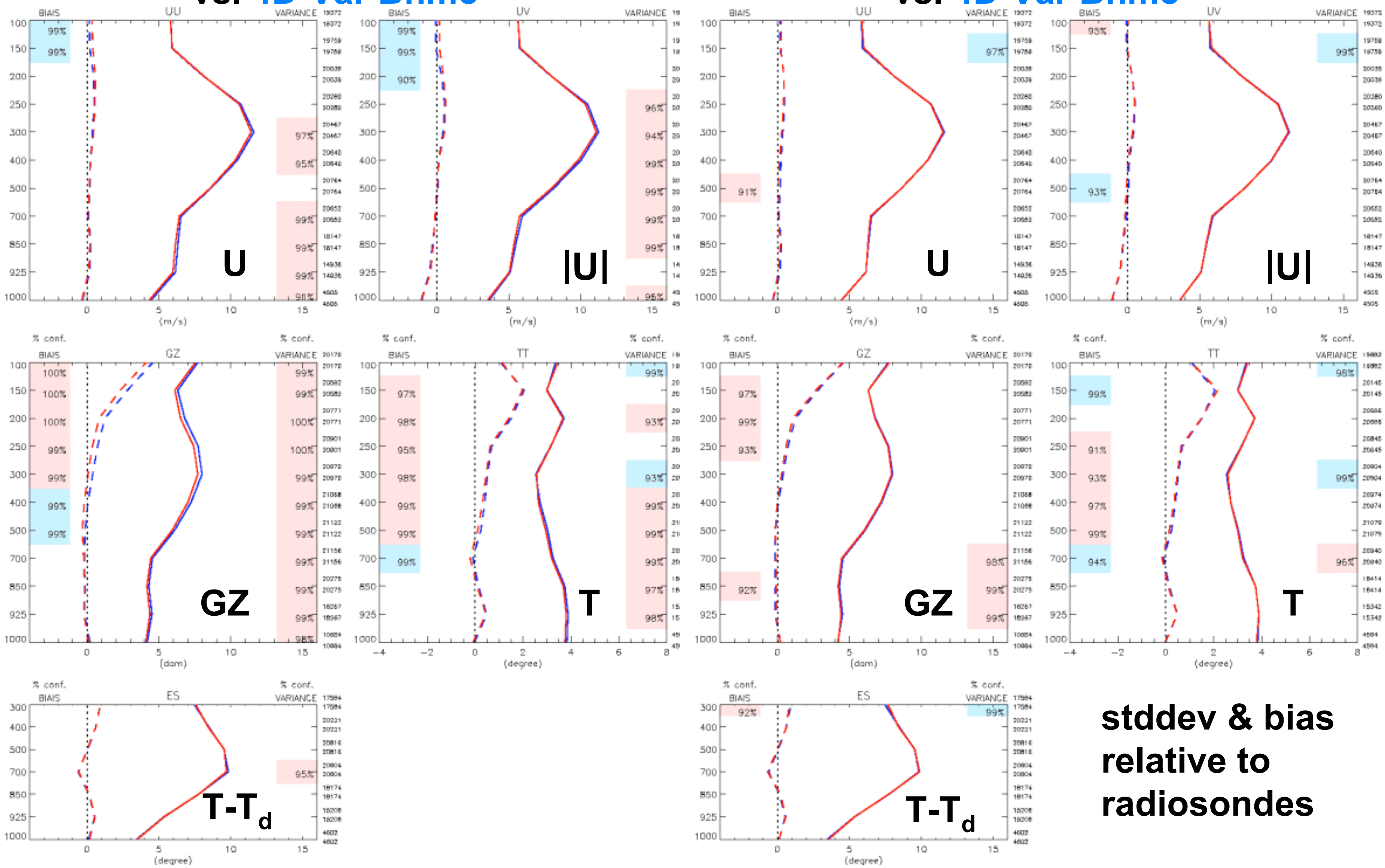


Forecast Results – 120h NH

EnKF mean analysis vs. 4D-Var Bnmc

4D-Var Benkf vs. 4D-Var Bnmc

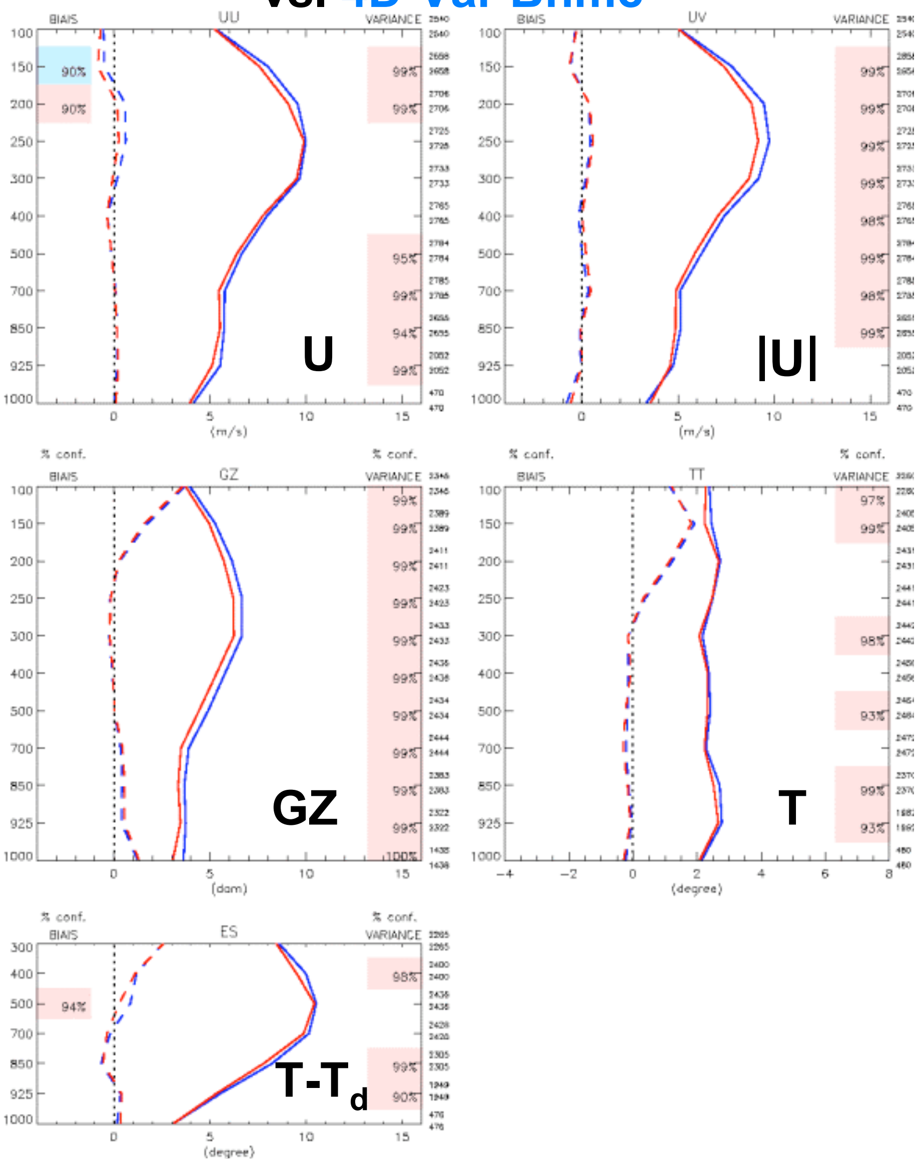
HYBRID



stddev & bias relative to radiosondes

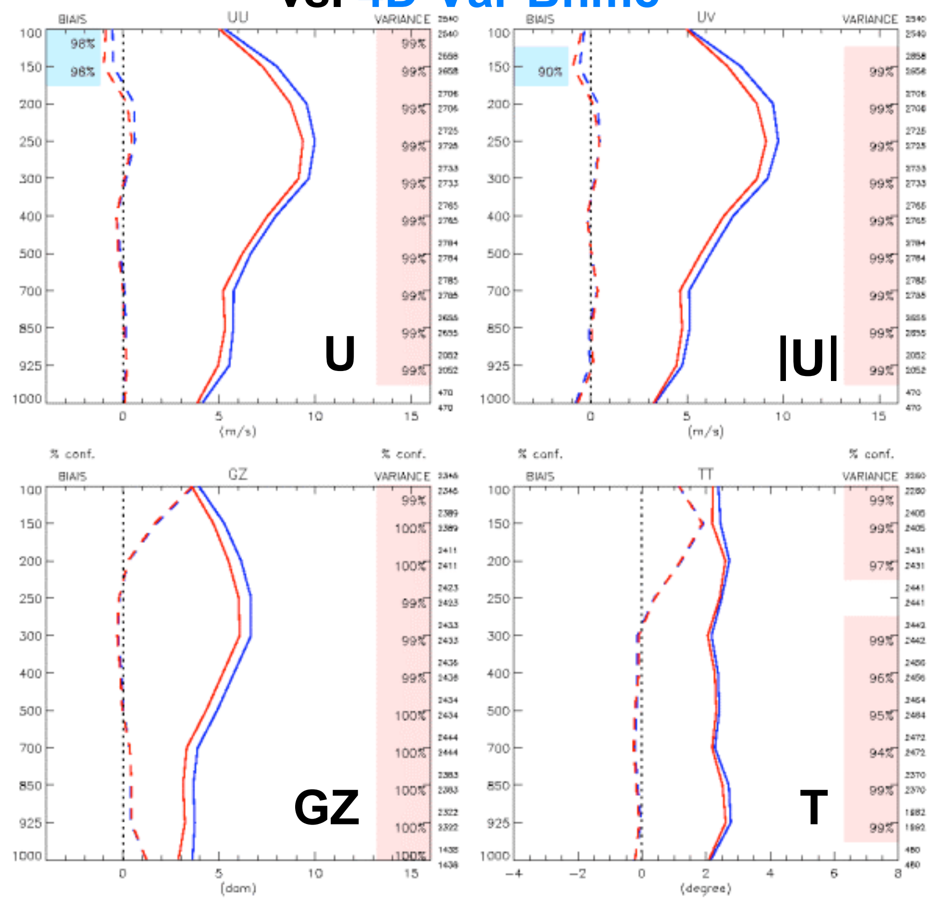
Forecast Results – 120h SH

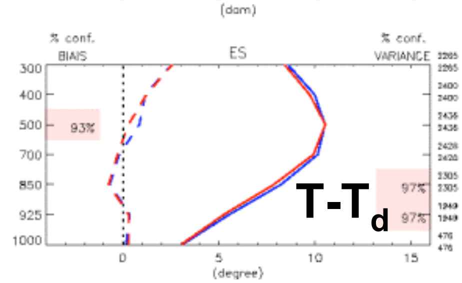
EnKF mean analysis vs. 4D-Var Bnmc



4D-Var Benkf vs. 4D-Var Bnmc

HYBRID





stddev & bias relative to radiosondes

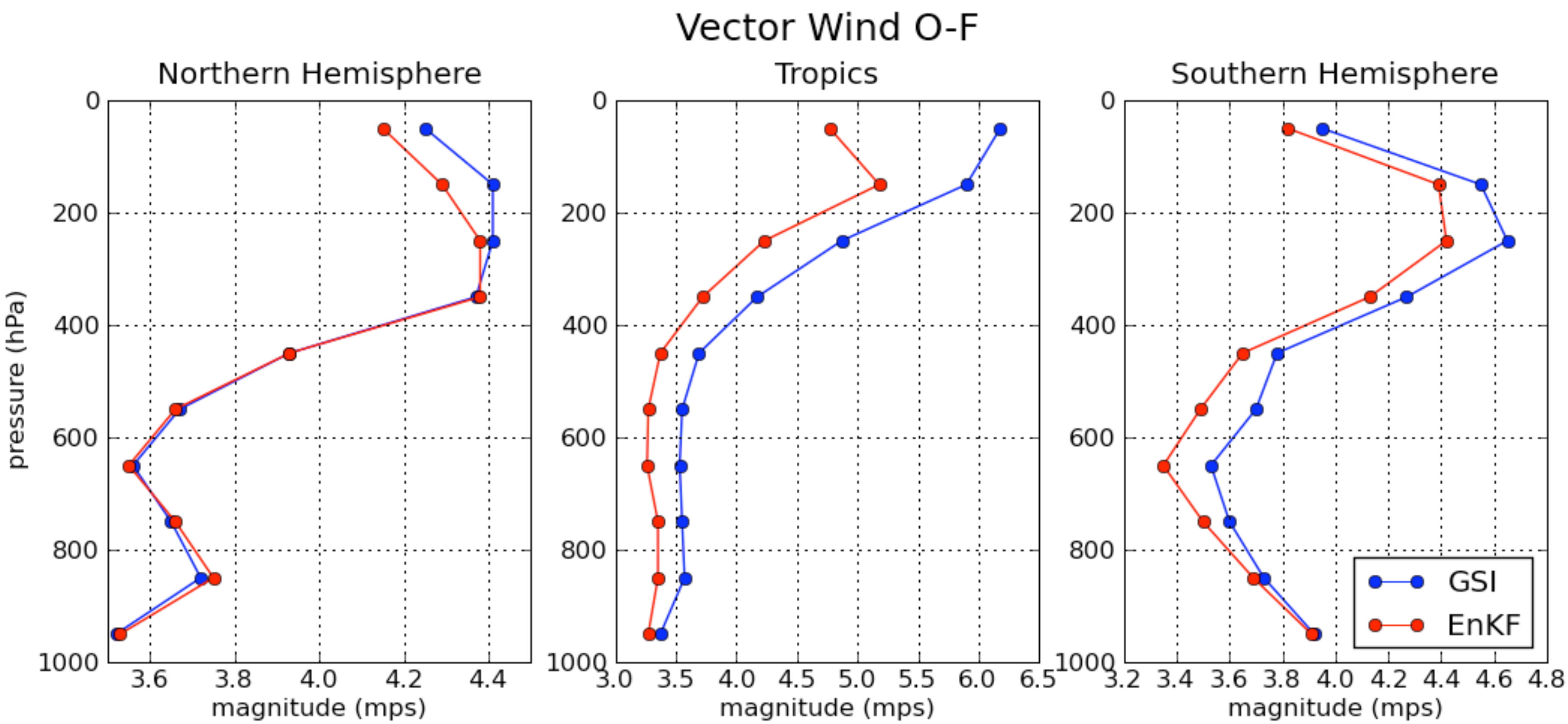
Conclusions from a clean intercomparison of 4D-Var and EnKF (Buehner et al., Canada, 2008)

✓ When running with the same (inner loop) model, same observations the forecast scores of 4D-Var and EnKF, are essentially identical (February 2007).

✓ When B_{NMC} in 4D-Var replaced by B_{EnKF} 4D-Var AC improved in the SH by 10 hours

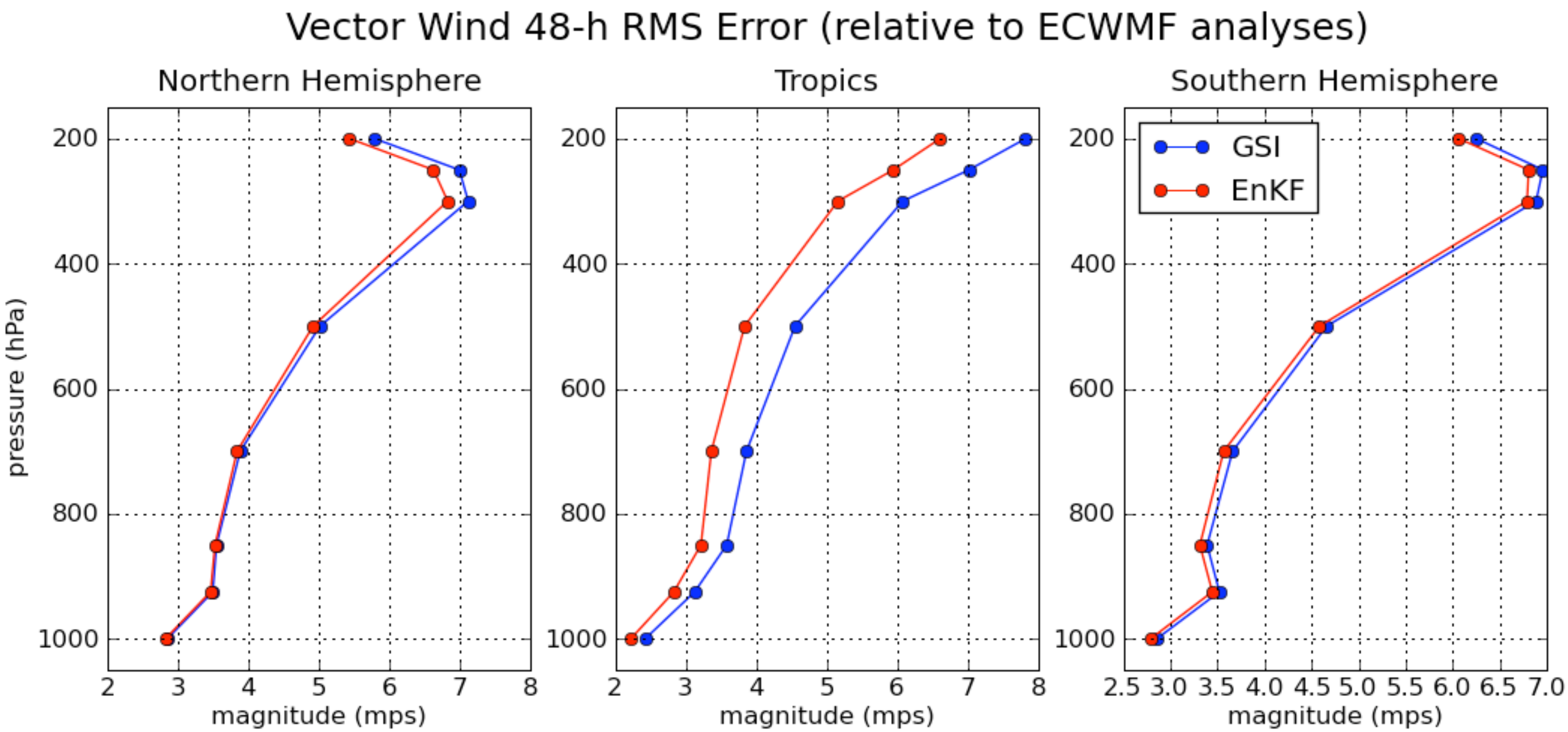
✓ They will run an incremental EnKF (hi-res) so the control models have same resolution!

Whitaker: Wind O-F 3-9hr statistics (all in situ data aggregated in 100 mb layers, 20071208-20080131)



- EnKF significantly better in Tropics, SH above boundary layer, NH upper trop and strat.

Whitaker: 48-h wind forecasts verified against operational ECMWF analyses



Conclusions from a clean comparison of GSI and EnKF (Whitaker, Dec 08)

- ✓ At T126/L64 resolution - 64 members - EnKF is clearly better than the operational GSI (same resolution) and it now takes only 4 times longer
- ✓ Will test incremental EnKF at T382/L64

Conclusions from a clean comparison of JMA 4D-Var and LETKF (Miyoshi et al. 08)

- ✓ At the same resolution LETKF is faster than the operational 4D-Var, better in the tropics and NH, worse in SH due to a model bias
- ✓ Will test simple low-dim method to correct model bias

Some ideas to improve LETKF/EnKF

We can adapt ideas developed within 4D-Var:

- ✓ **No-cost smoother** (Kalnay et al, Tellus 2007)
- ✓ "**Outer loop**" and nonlinearities (Yang and Kalnay)
- ✓ Accelerating the **spin-up** (Kalnay and Yang, QJ, subm.)
- ✓ Forecast **sensitivity** to observations (Liu and Kalnay, QJ, 2008)
- ✓ **Coarse** analysis resolution without degradation (Yang, Kalnay, Hunt, Bowler, QJ, in press)
- ✓ Low-dimensional **model bias correction** (Li, Kalnay, Danforth, Miyoshi, MWR, submitted)
- ✓ Simultaneous estimation of **optimal inflation** and **observation errors** (Li, Kalnay, and Miyoshi, QJ, in press).

Some ideas to improve LETKF/EnKF

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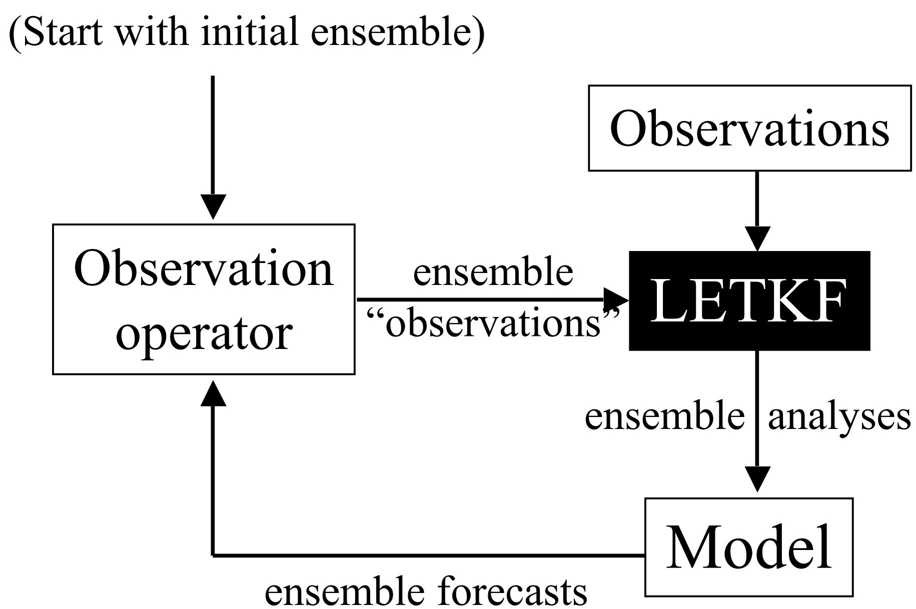
EnKF is new, simple, flexible, and there is a whole community eager to test new ideas

Local Ensemble Transform Kalman Filter (Ott et al, 2004, Hunt et al, 2004, 2007)

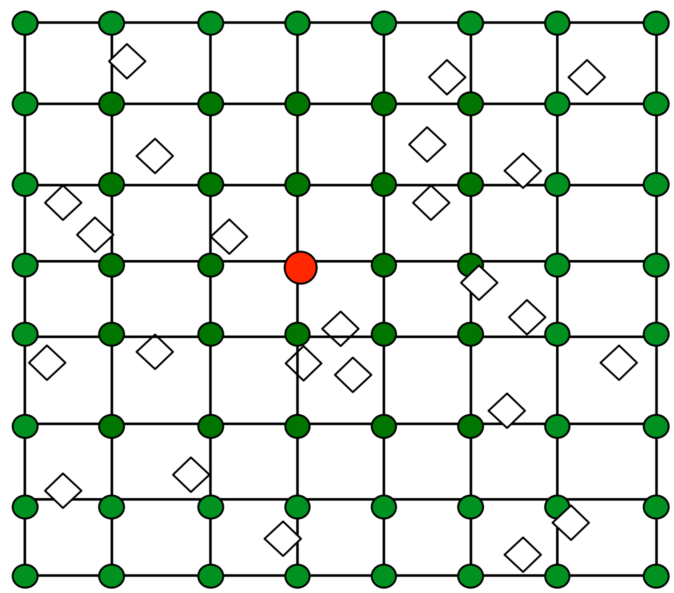


- Model independent (black box)
- Obs. assimilated simultaneously at each grid point
- 100% parallel: very fast
- No **adjoint** needed
- **4D LETKF extension**

Localization based on observations

Perform data assimilation in a local volume, choosing observations

The state estimate is updated at the central grid red dot

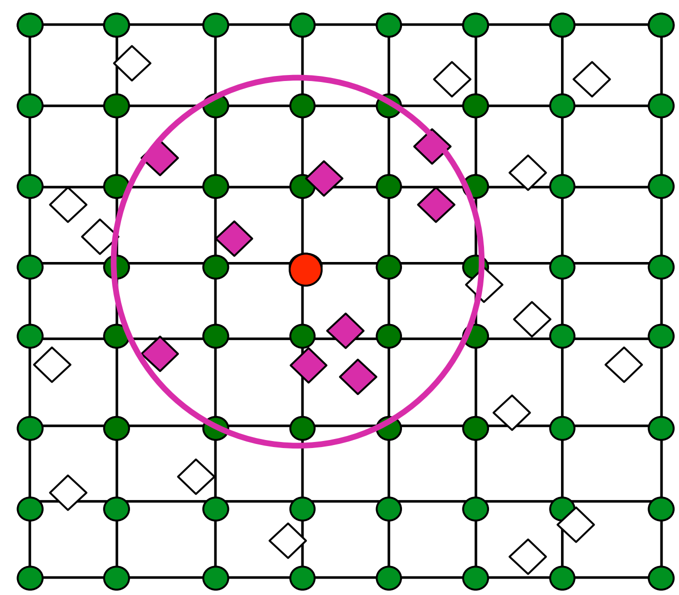


Localization based on observations

Perform data assimilation in a local volume, choosing observations

The state estimate is updated at the central grid red dot

All observations (purple diamonds) within the local region are assimilated



The LETKF algorithm can be described **in a single slide**!

Local Ensemble Transform Kalman Filter (LETKF)

Globally:

Forecast step: $\mathbf{x}_{n,k}^b = M_n\left(\mathbf{x}_{n-1,k}^a\right)$

Analysis step: construct $\mathbf{X}^b = \left[\mathbf{x}_1^b - \overline{\mathbf{x}}^b \mid \ldots \mid \mathbf{x}_K^b - \overline{\mathbf{x}}^b\right];$

$$\mathbf{y}_i^b = H(\mathbf{x}_i^b);\ \mathbf{Y}_n^b = \left[\mathbf{y}_1^b - \overline{\mathbf{y}}^b \mid \ldots \mid \mathbf{y}_K^b - \overline{\mathbf{y}}^b\right]$$

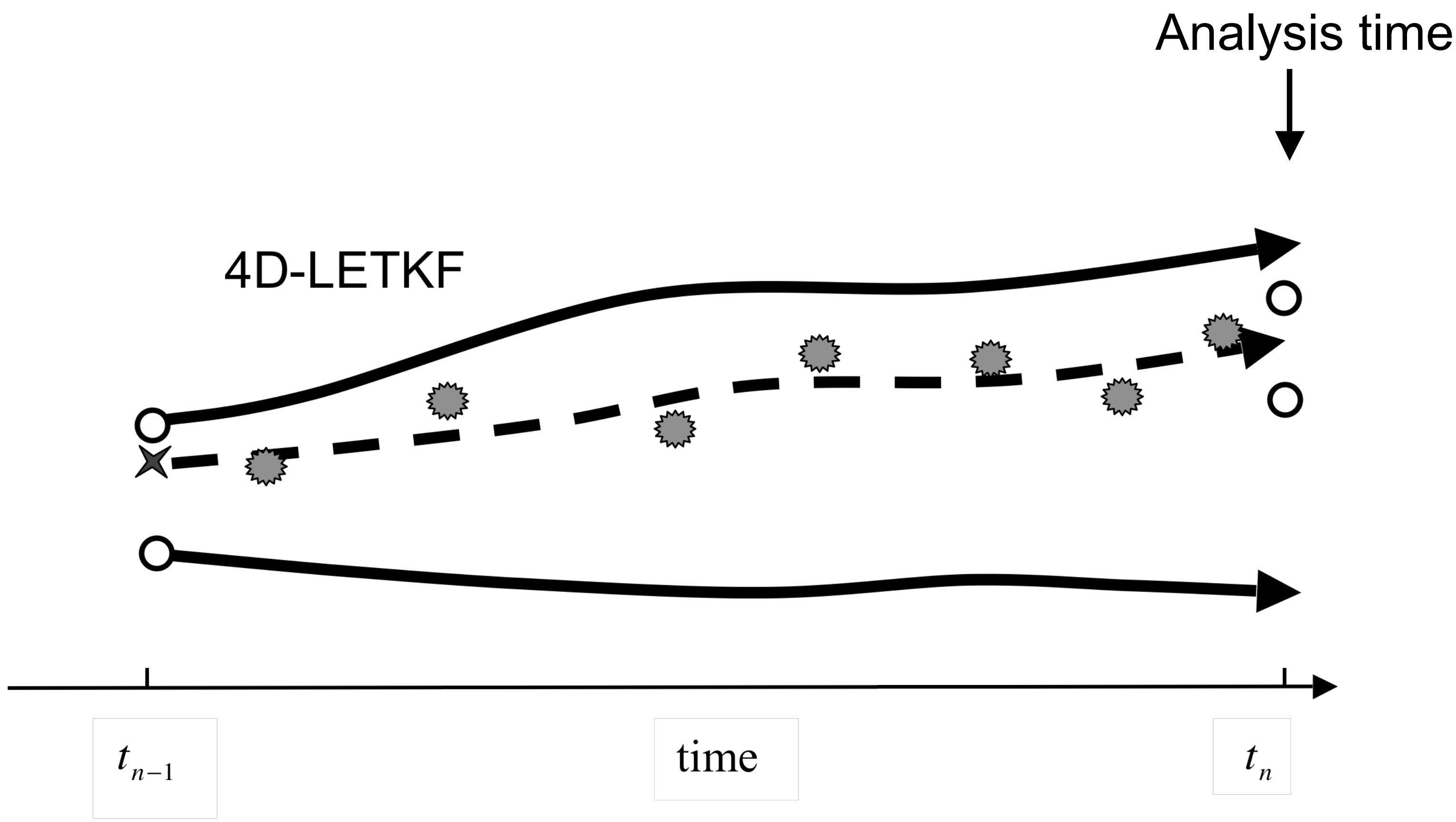
Locally: Choose for each grid point the observations to be used, and compute the local analysis error covariance and perturbations in ensemble space:

$$\tilde{\mathbf{P}}^a = \left[\left(K-1\right)\mathbf{I} + \mathbf{Y}^{bT}\mathbf{R}^{-1}\mathbf{Y}^b\right]^{-1};\ \mathbf{W}^a = [(K-1)\tilde{\mathbf{P}}^a]^{1/2}$$

Analysis mean in ensemble space: $\overline{\mathbf{w}}^a = \tilde{\mathbf{P}}^a\mathbf{Y}^{bT}\mathbf{R}^{-1}(\mathbf{y}^o - \overline{\mathbf{y}}^b)$

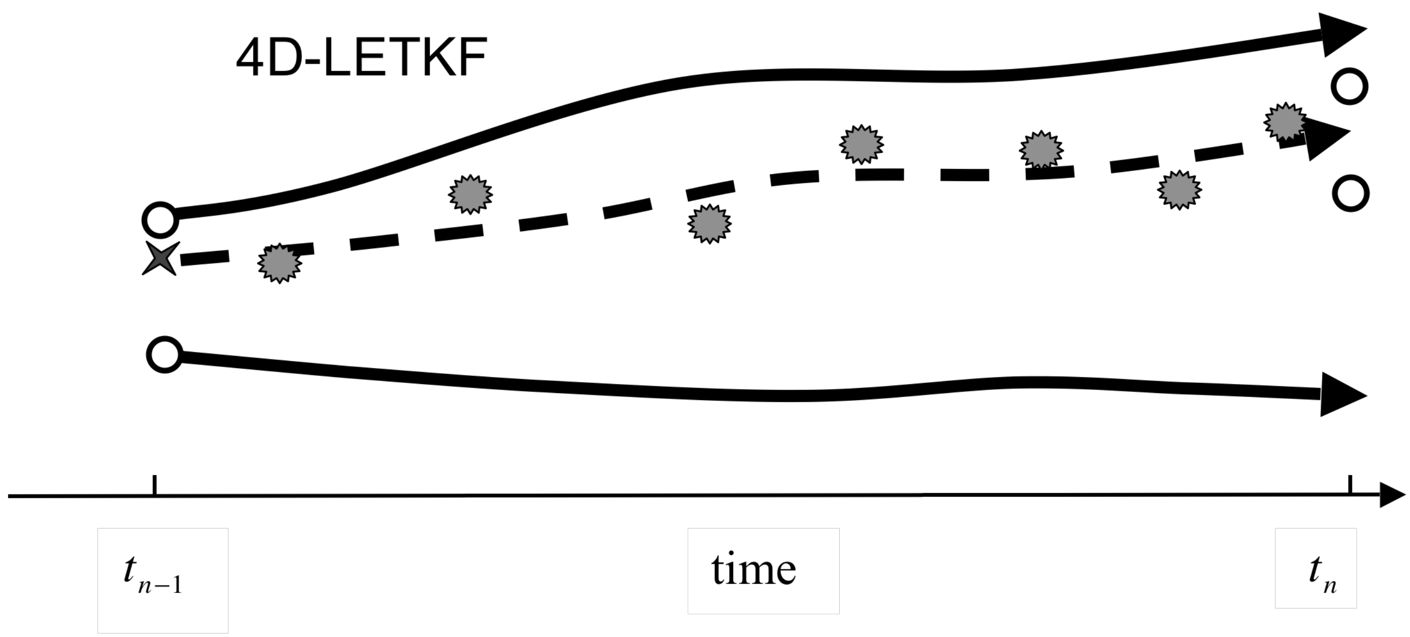
and add to $\mathbf{W}^a$ to get the analysis ensemble in ensemble space

The new ensemble analyses in model space are the columns of $\mathbf{X}_n^a = \mathbf{X}_n^b\mathbf{W}^a + \overline{\mathbf{x}}^b$. Gathering the grid point analyses forms the new global analyses. Note that the the output of the LETKF are analysis weights $\overline{\mathbf{w}}^a$ and perturbation analysis matrices of weights $\mathbf{W}^a$. These weights multiply the ensemble forecasts.



The 4D-LETKF produces an analysis in terms of weights of the ensemble forecast members at the analysis time t_n, giving the trajectory that best fits all the observations in the assimilation window.

No-cost LETKF smoother ($\times$): apply at t_{n-1} the same weights found optimal at t_n. It works for 3D- or 4D-LETKF



We can get the smoothed mean analysis (used for the **outer loop**) and the smoothed analysis error covariance (used for "**Running in Place**" to deal with spin-up)

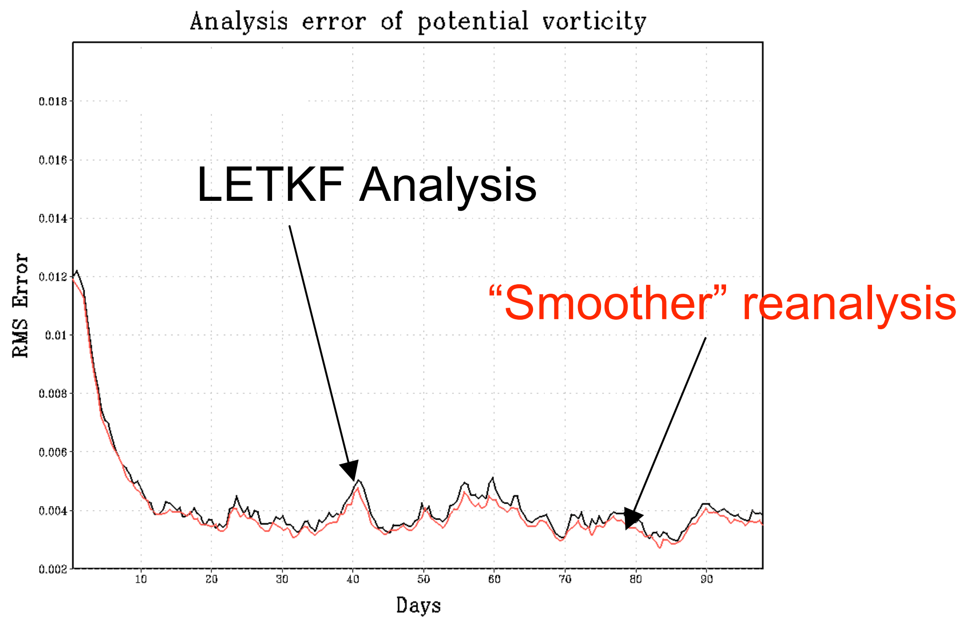
No-cost LETKF smoother test on a QG model: It works!

LETKF analysis
at time n

$$\overline{\mathbf{x}}_n^a = \overline{\mathbf{x}}_n^f + \mathbf{X}_n^f \overline{\mathbf{w}}_n^a$$

Smoother analysis
at time n-1

$$\tilde{\mathbf{x}}_{n-1}^a = \overline{\mathbf{x}}_{n-1}^f + \mathbf{X}_{n-1}^f \overline{\mathbf{w}}_n^a$$



This very simple smoother allows us to go back and forth in time within an assimilation window:
it allows assimilation of **future** data in reanalysis

Nonlinearities and "outer loop"

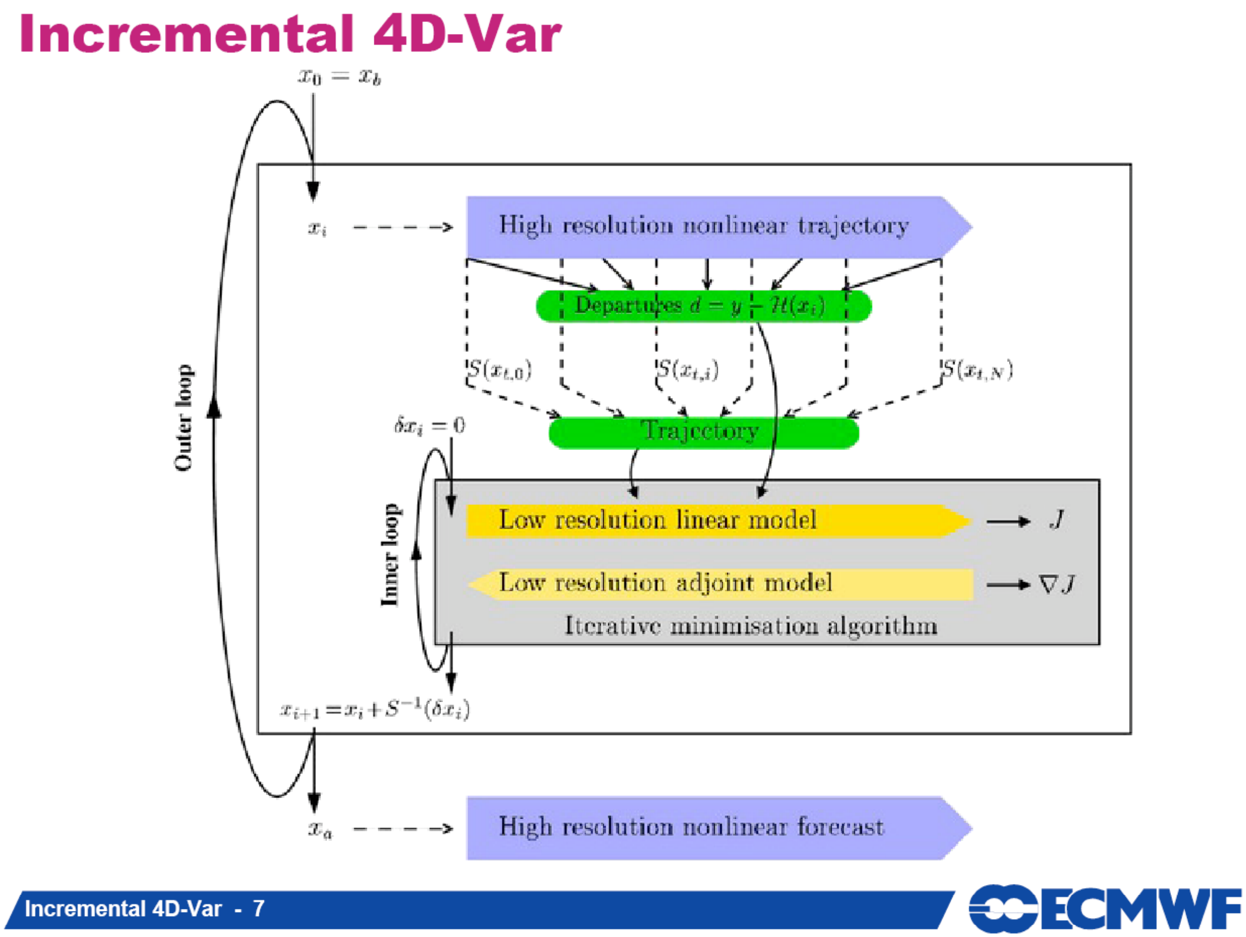
- The main disadvantage of EnKF is that it cannot handle nonlinear (non-Gaussian) perturbations and therefore needs short assimilation windows.
- It doesn't have the important outer loop so important in 3D-Var and 4D-Var (DaSilva, pers. comm. 2006)

Lorenz -3 variable model (Kalnay et al. 2007a Tellus), RMS analysis error

	4D-Var	LETKF	
Window=8 steps	0.31	**0.30**	(linear window)
Window=25 steps	**0.53**	0.66	(nonlinear window)

Long windows + Pires et al. => 4D-Var clearly wins!

"Outer loop" in 4D-Var



Nonlinearities and “outer loop”

Outer loop: similar to 4D-Var: use the final weights to correct only the mean initial analysis, keeping the initial perturbations. Repeat the analysis once or twice. It centers the ensemble on a more accurate nonlinear solution.

Miyoshi: Jaszwinski (1970) also suggested this “inner loop” in a footnote!

Lorenz -3 variable model RMS analysis error

	4D-Var	LETKF	LETKF +outer loop
Window=8 steps	0.31	0.30	**0.27**
Window=25 steps	0.53	0.66	**0.48**

“Running in place” further reduces RMS from 0.48 to 0.39!

“Running in place” to spin-up faster

Kalnay and Yang (2008)

- 4D-Var spins-up faster than EnKF because it is a smoother: it keeps iterating until it fits the observations within the assimilation window as well as possible
- EnKF spins-up fast if starting from a “good” initial state, e.g., 3D-Var, but needs also an ensemble representing the “errors of the day”
- In a severe storm where radar observations start with the storm, there is little real time to spin-up
- Caya et al. (2005): “EnKF is eventually better than 4D-Var” (but it is too late to be useful, it misses the storm).
- Jidong Gao, (pers. comm. 2007): spin-up is the main obstacle for the use of EnKF for storm prediction.

Can we use the data more than once?

- **Hunt et al., 2007**: The background term represents the evolution of the maximum likelihood trajectory given all the observations in the past

$$\sum_{j=1}^{n-1}\left[\mathbf{y}_j^o-\mathbf{H}_j\mathbf{M}_{t_n,t_j}\mathbf{x}\right]^T\mathbf{R}_j^{-1}\left[\mathbf{y}_j^o-\mathbf{H}_j\mathbf{M}_{t_n,t_j}\mathbf{x}\right]=\left[\mathbf{x}-\overline{\mathbf{x}}_n^b\right]^T\left(\mathbf{P}_n^b\right)^{-1}\left[\mathbf{x}-\overline{\mathbf{x}}_n^b\right]+c$$

- After the analysis a similar relationship is valid:

$$\left[\mathbf{x}-\overline{\mathbf{x}}_n^b\right]^T\left(\mathbf{P}_n^b\right)^{-1}\left[\mathbf{x}-\overline{\mathbf{x}}_n^b\right]+\left[\mathbf{y}_n^o-\mathbf{H}_n\mathbf{x}\right]^T\left(\mathbf{R}_n^{-1}\right)\left[\mathbf{y}_n^o-\mathbf{H}_n\mathbf{x}\right]=\left[\mathbf{x}-\overline{\mathbf{x}}_n^a\right]^T\left(\mathbf{P}_n^a\right)^{-1}\left[\mathbf{x}-\overline{\mathbf{x}}_n^a\right]+c'$$

- From here one can derive the linear KF equations
- Also the rule: “Use the data once and then discard it”

"Running in Place": like the outer loop but updating also the covariance

- EnKF is a sequential data assimilation system where, after the new data is used at the analysis time, it should be discarded…

- **only** if the previous analysis and the new background are the most likely states given the past observations.

- **If the system has converged after the initial spin-up all the information from past observations is already included in the background.**

- **During spin-up we should use the observations repeatedly if we can extract extra information**. But we should avoid overfitting the observations

Running in Place algorithm

Cold-start the EnKF from any initial ensemble mean and random perturbations at t_0, and integrate the initial ensemble to t_1. The "running in place" loop with $n=1$, is:

a) Perform a standard EnKF analysis and obtain the analysis weights at t_n, saving the mean square observations minus forecast (OMF) computed by the EnKF.

b) Apply the no-cost smoother to obtain the smoothed analysis ensemble at t_{n-1} by using the same weights obtained at t_n.

c) Perturb the smoothed analysis ensemble with a small amount of random Gaussian perturbations, similar to additive inflation.

d) Integrate the perturbed smoothed ensemble to t_n. If the forecast fit to the observations is smaller than in the previous iteration according to some criterion, go to a) and perform another iteration. If not, let $t_{n-1} \leftarrow t_n$ and proceed to the next assimilation window.

Running in Place algorithm (notes)

Notes:

c) *Perturb the smoothed analysis ensemble with a small amount of random Gaussian perturbations, a method similar to additive inflation.*

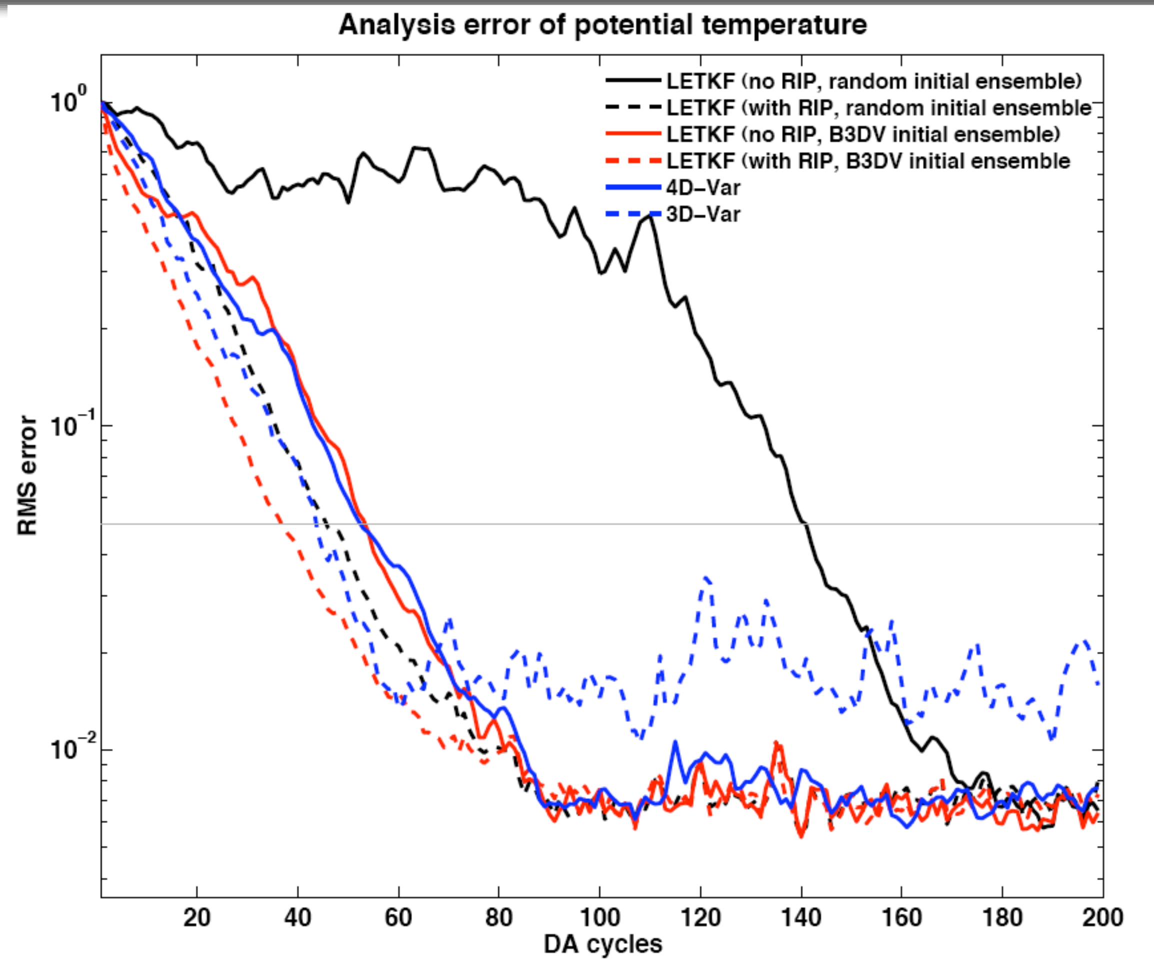
This perturbation has two purposes:

1) Avoid reaching the same analysis as before, and
2) Encourage the ensemble to explore new unstable directions

d) *Convergence criterion*: if $$\frac{OMF^2(iter) - OMF^2(iter+1)}{OMF^2(iter)} > \varepsilon$$

with $\varepsilon \sim 5\%$ do another iteration. Otherwise go to the next assimilation window.

Results with a QG model



Spin-up depends on initial perturbations, but RIP works well even with random perturbations. It becomes as fast as 4D-Var (blue). RIP takes only 2-4 iterations.

Results with a QG model

	LETKF Random initial ensemble		LETKF B3DV initial ensemble		LETKF, Random initial ensemble	Variational	
	No RIP	With RIP	No RIP	With RIP	Fixed 10 iterations RIP	3D-Var B3DV	4D-Var 0.05B3DV
Spin-up: DA cycles to reach 5% error	**141**	**46**	**54**	**37**	**37**	**44**	**54**
RMS error ($x10^{-2}$)	0.5	0.54	0.5	0.52	1.16	1.24	0.54

LETKF spin-up from random perturbations: 141 cycles. With RIP: 46 cycles

LETKF spin-up from 3D-Var perts. 54 cycles. With RIP: 37 cycles

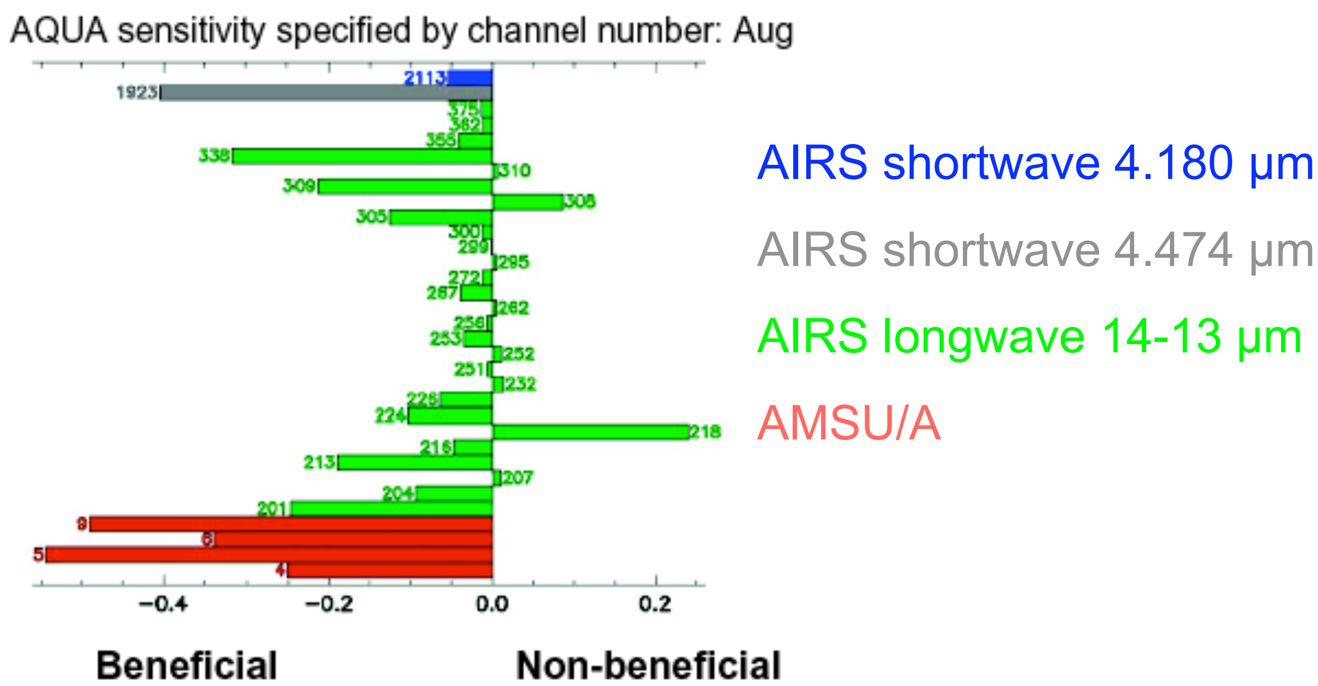
4D-Var spin-up using 3D-Var prior: 54 cycles.

Estimation of forecast sensitivity to observations without adjoint in an ensemble Kalman filter

Junjie Liu and Eugenia Kalnay

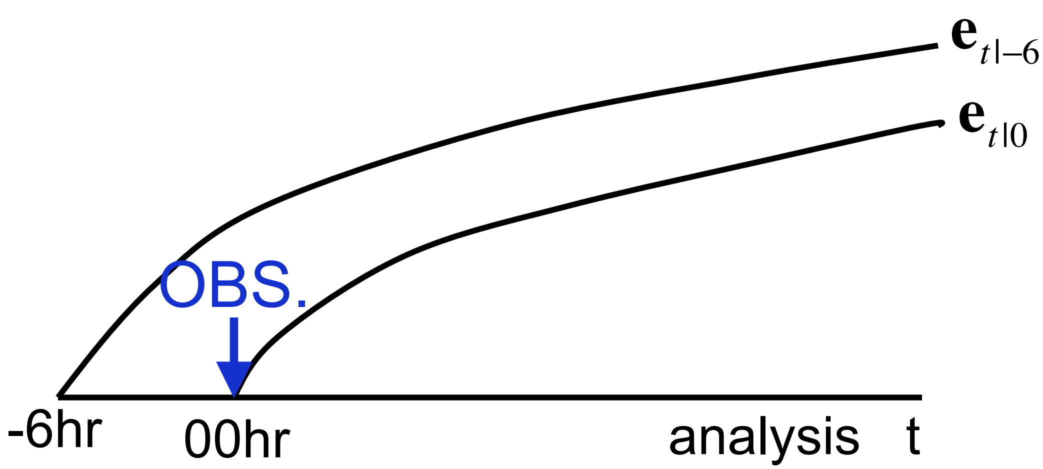
QJRMS October 2008

Motivation: Langland and Baker (2004)



- The adjoint method proposed by Langland and Baker (2004) and Zhu and Gelaro (2007) quantifies the reduction in forecast error for each individual observation source
- The adjoint method detects the observations which make the forecast worse.
- The adjoint method requires adjoint model which is difficult to get.

Schematic of the observation impact on the reduction of forecast error



$$\mathbf{e}_{t|-6} = \overline{\mathbf{x}}^f_{t|-6} - \overline{\mathbf{x}}^a_t$$

$$\mathbf{e}_{t|0} = \overline{\mathbf{x}}^f_{t|0} - \overline{\mathbf{x}}^a_t$$

(Adapted from Langland and Baker, 2004)

The only difference between $\mathbf{e}_{t|0}$ and $\mathbf{e}_{t|-6}$ is the assimilation of observations at 00hr.

- Observation impact on the reduction of forecast error: $J = \frac{1}{2}(\mathbf{e}^T_{t|0}\mathbf{e}_{t|0} - \mathbf{e}^T_{t|-6}\mathbf{e}_{t|-6})$

The ensemble sensitivity method

Euclidian cost function: $$J = \frac{1}{2}(\mathbf{e}_{t|0}^T \mathbf{e}_{t|0} - \mathbf{e}_{t|-6}^T \mathbf{e}_{t|-6}) \qquad \mathbf{v}_0 = \mathbf{y}_0^o - h(\overline{\mathbf{x}}_{0|-6}^b)$$

Cost function as function of obs. Increments: $$J = \left\langle \mathbf{v}_0, \frac{\partial J}{\partial \mathbf{v}_0} \right\rangle$$

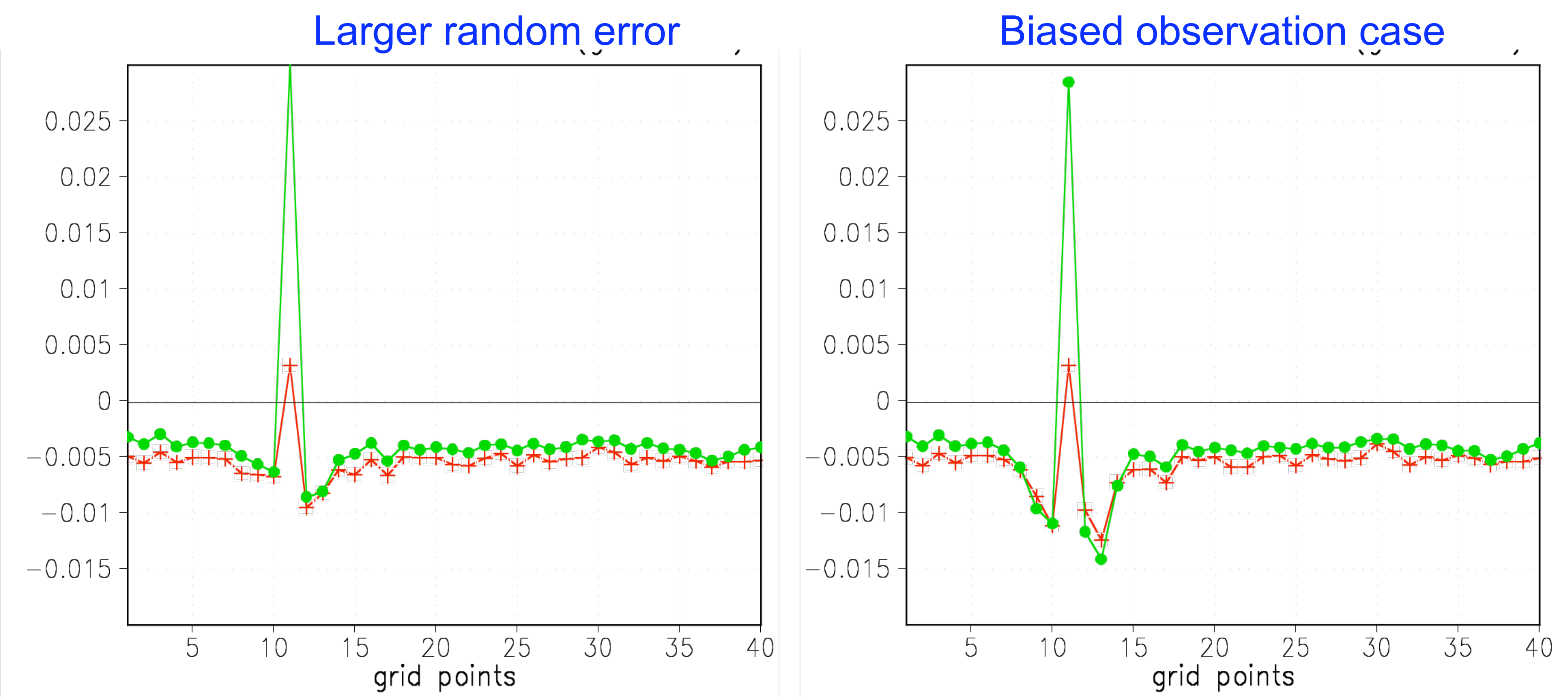
The sensitivity of cost function with respect to the assimilated observations:

$$\frac{\partial J}{\partial \mathbf{v}_0} = \left[\tilde{\mathbf{K}}_0^T \mathbf{X}_{t|-6}^{fT} \right] \left[\mathbf{e}_{t|-6} + \mathbf{X}_{t|-6}^f \tilde{\mathbf{K}}_0 \mathbf{v}_0 \right]$$

With this formula we can predict the impact of observations on the forecasts!

Test ability to detect the poor quality observation on the Lorenz 40 variable model

Observation impact from LB (red) and from ensemble sensitivity method (green)



✓ Like adjoint method, ensemble sensitivity method can detect the observation poor quality (11th observation location)

✓ The ensemble sensitivity method has a stronger signal when the observation has negative impact on the forecast.

Summary for forecast sensitivity to obs.

- Derived a formula to calculate the observation impact based on the ensemble without using the adjoint model which usually is not available.

- The results based on Lorenz-40 variable model show that ensemble sensitivity method without using adjoint model gives results similar to adjoint method .

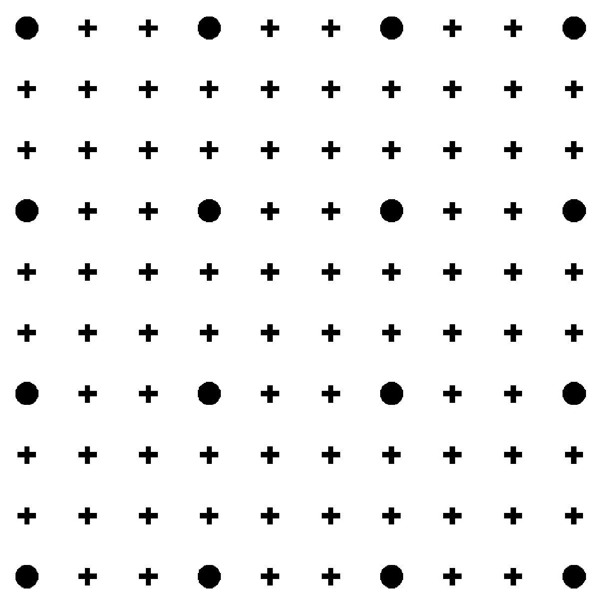
- Like adjoint method, ensemble sensitivity method can detect the observation which either has larger random error or has bias. Under such conditions, the ensemble sensitivity method has stronger and more accurate signal.

- It provides a powerful tool to check the quality of the observations.

Coarse analysis with interpolated weights

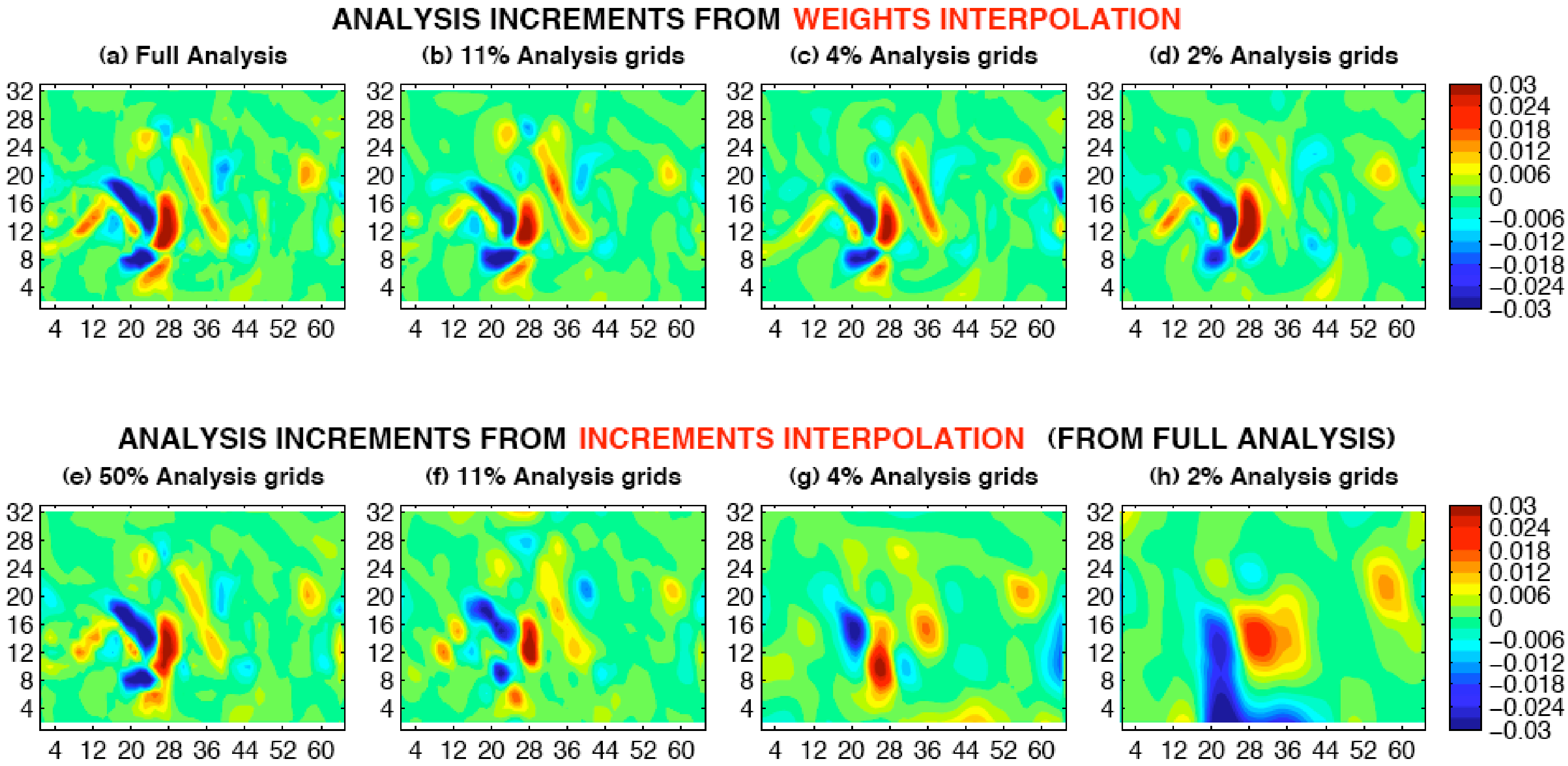
Yang et al (2008)

- In EnKF the analysis is a weighted average of the forecast ensemble
- We performed experiments with a QG model **interpolating <u>weights</u>** compared to **analysis <u>increments</u>**.
- Coarse grids of 11%, 4% and 2% interpolated analysis points.
- **Weight fields vary on large scales: they interpolate very well**



1/(3x3)=11% analysis grid

Weight interpolation versus **Increment** interpolation

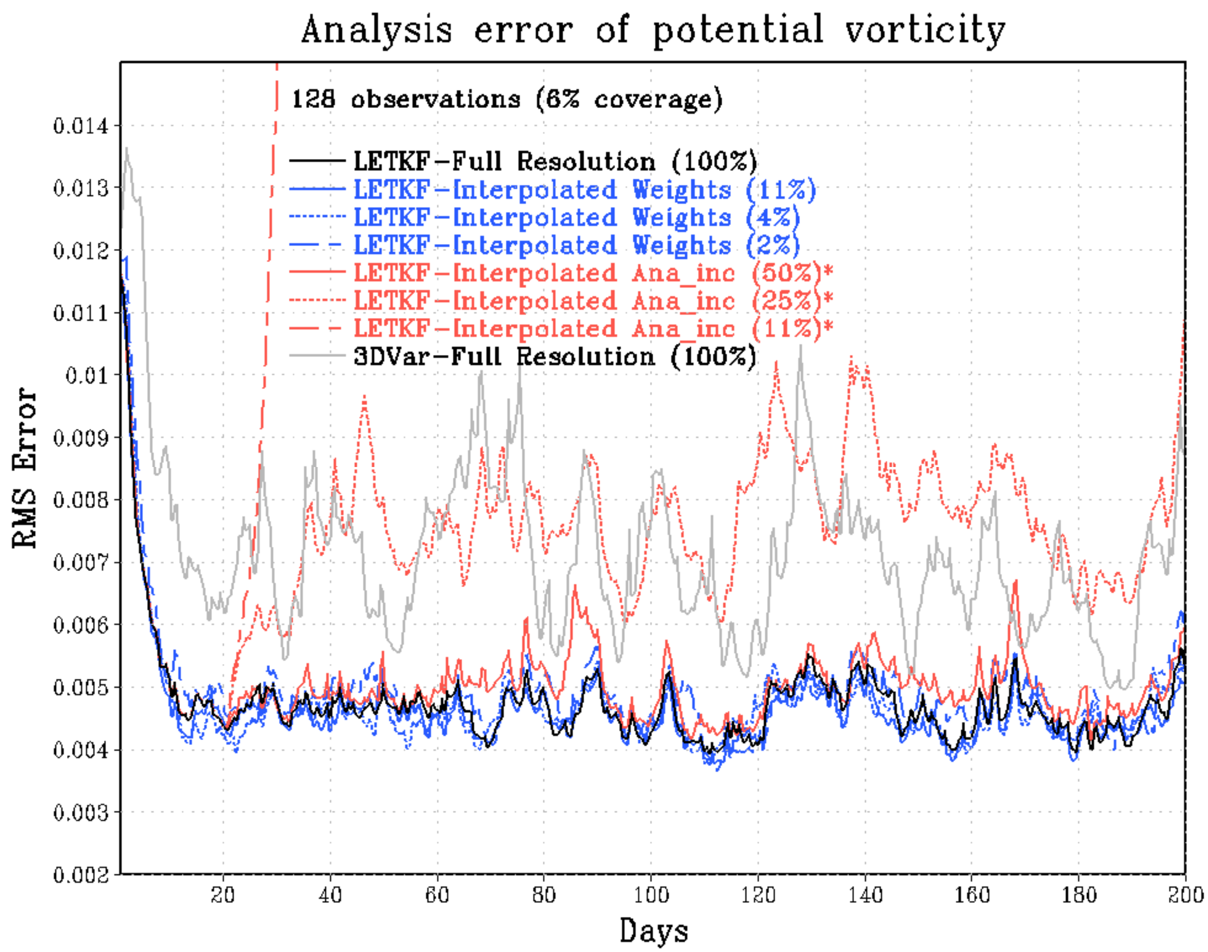


With increment interpolation, the analysis is OK only with 50% analysis coverage

With weight interpolation, there is almost no degradation!

LETKF maintains balance and conservation properties

Impact of coarse analysis on accuracy



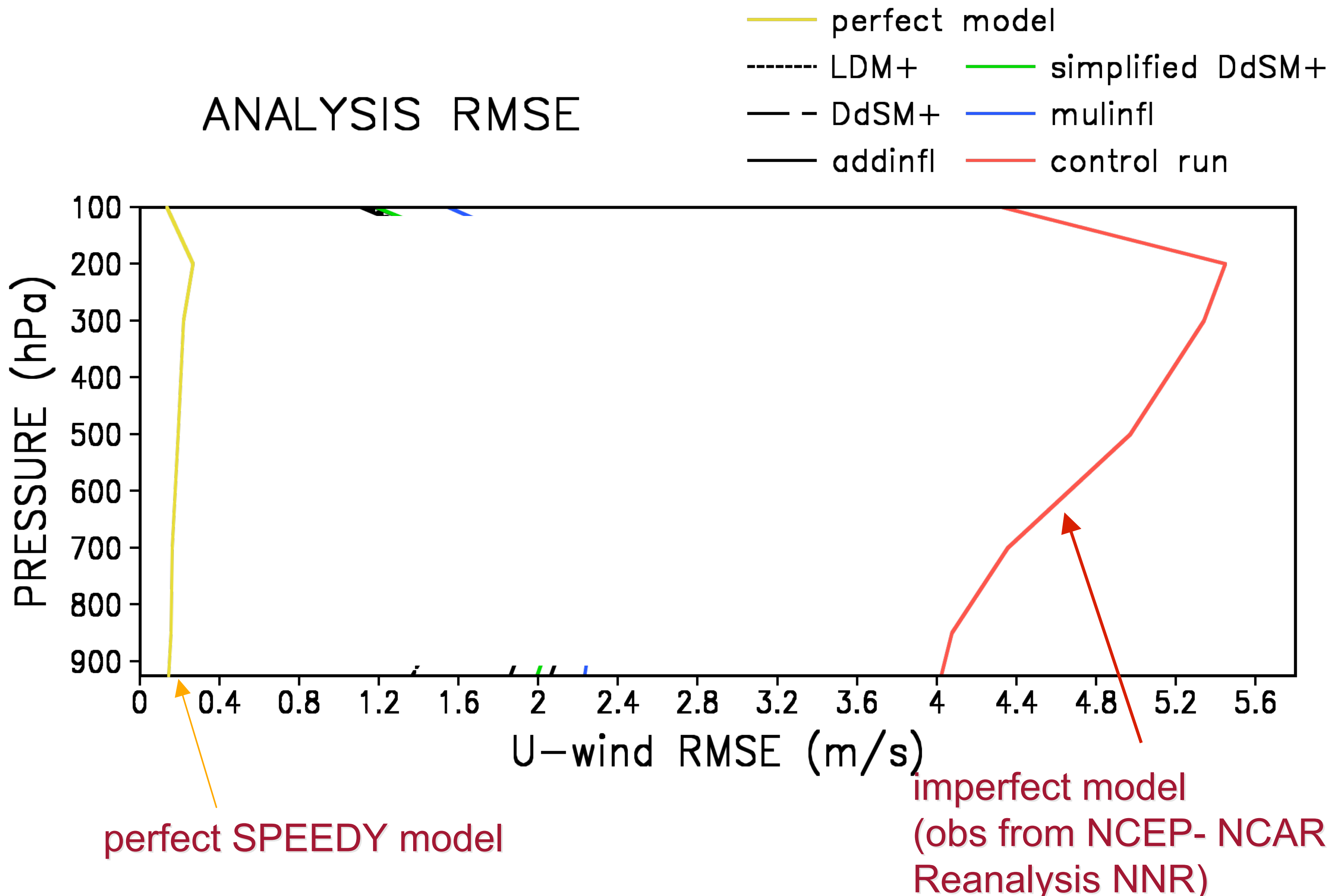
With increment interpolation, the analysis degrades

With weight interpolation, there is no degradation, the analysis is actually better!

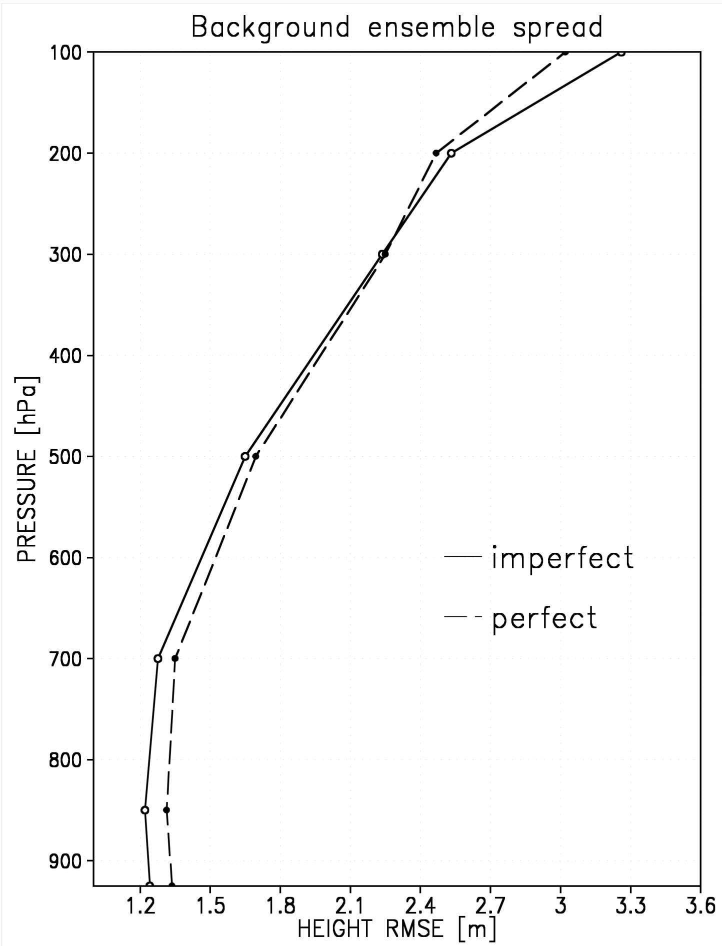
Model error: comparison of methods to correct model bias and inflation

Hong Li, Chris Danforth, Takemasa Miyoshi, and Eugenia Kalnay. QJ (in press)

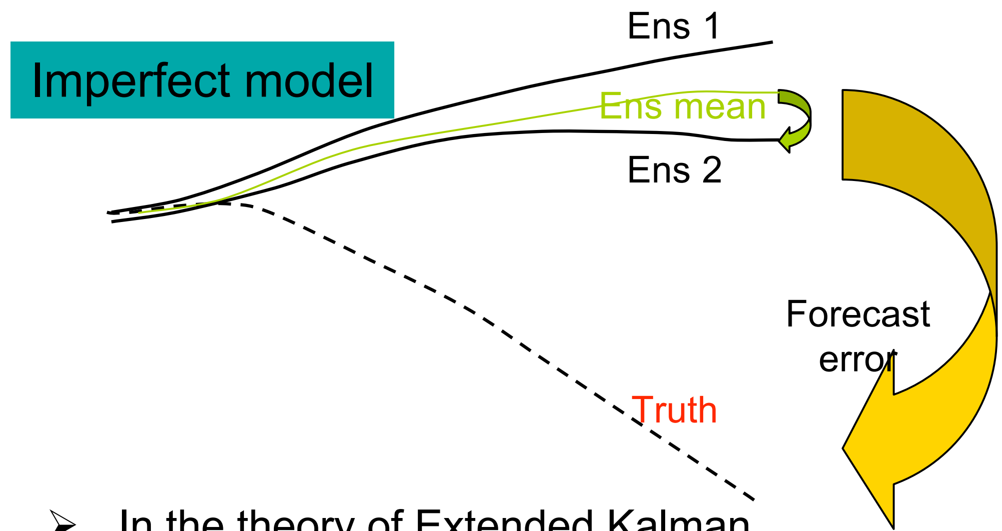
Model error: If we assume a perfect model in EnKF, we underestimate the analysis errors (Li, 2007)



— Why is EnKF vulnerable to model errors ?



The ensemble spread is 'blind' to model errors



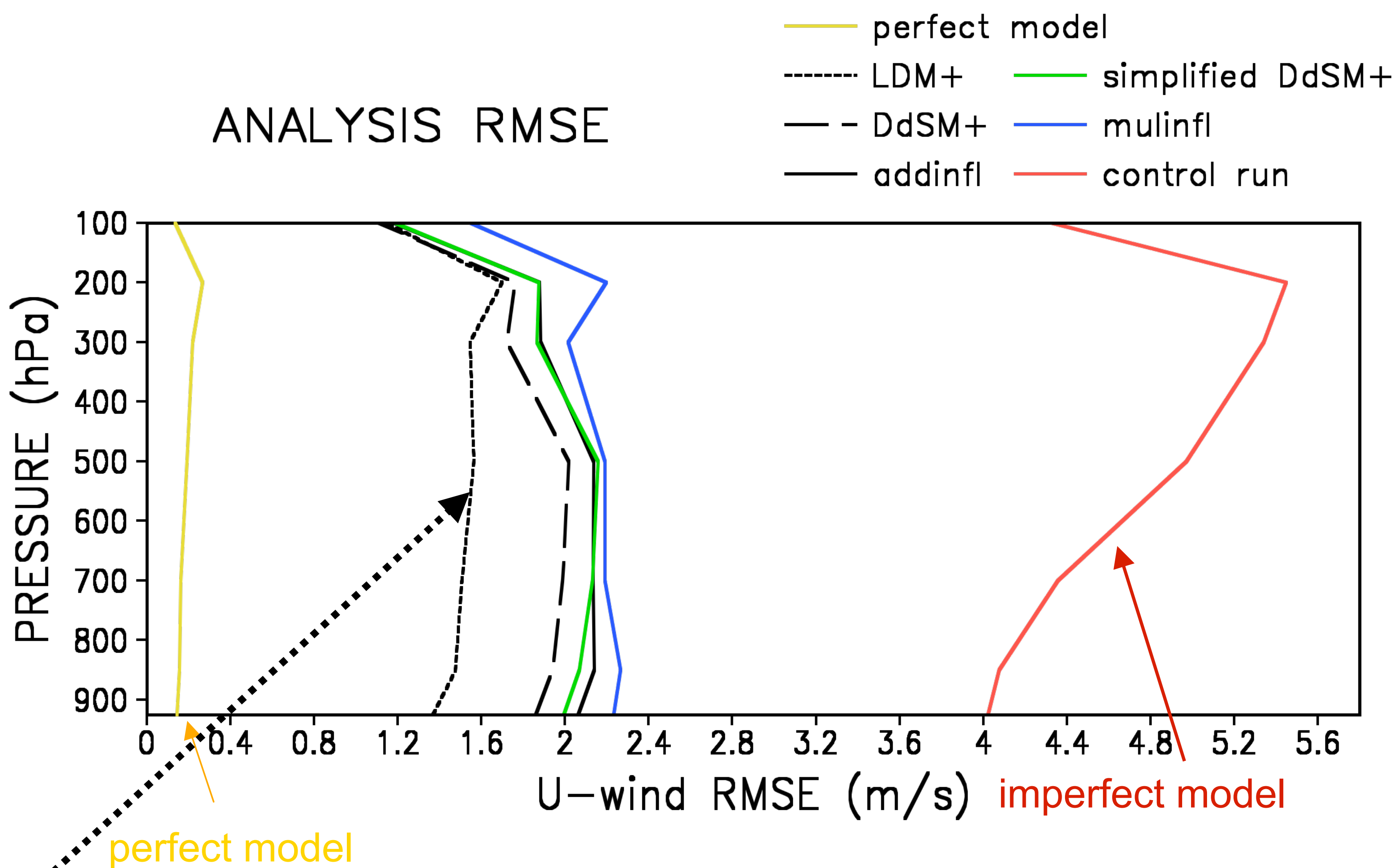
- In the theory of Extended Kalman filter, forecast error is represented by the growth of errors in IC and the model errors.

$$\mathbf{P}_i^f = \mathbf{M}_{\mathbf{x}_{i-1}^a} \mathbf{P}_{i-1}^a \mathbf{M}_{\mathbf{x}_{i-1}^a}^T + \boxed{\mathbf{Q}}$$

- However, in ensemble Kalman filter, error estimated by the ensemble spread can only represent the first type of errors.

$$\mathbf{P}_i^f \approx \frac{1}{k-1} \sum_{i=1}^{K} (x_i^f - \overline{x^f})(x_i^f - \overline{x^f})^T$$

We compared several methods to handle bias and random model errors



Low Dimensional Method to correct the bias (Danforth et al, 2007) combined with additive inflation

Simultaneous estimation of EnKF inflation and obs errors in the presence of model errors

Hong Li, Miyoshi and Kalnay (QJ, in press)

- Any data assimilation scheme requires accurate statistics for the observation and background errors (usually tuned or from gut feeling).
- EnKF needs inflation of the background error covariance: tuning is expensive
- Wang and Bishop (2003) and Miyoshi (2005) proposed a technique to estimate the covariance inflation parameter online. It works well if ob errors are accurate.
- We introduce a method to simultaneously estimate ob errors and inflation.

- We test the method for a perfect model and in the presence of model random errors (it works very well) and model bias (not so well).

Diagnosis of observation error statistics

Houtekamer et al (2001) well known statistical relationship:

OMB*OMB $$< \mathbf{d}_{o-b}\mathbf{d}^T_{o-b} >= \mathbf{H}\mathbf{P}^b\mathbf{H}^T + \mathbf{R}$$

Desroziers et al, 2005, introduced two new statistical relationships:

OMA*OMB $$< \mathbf{d}_{o-a}\mathbf{d}^T_{o-b} >= \mathbf{R}$$

AMB*OMB $$< \mathbf{d}_{a-b}\mathbf{d}_{o-b}{}^T >= \mathbf{H}\mathbf{P}^b\mathbf{H}^T$$

These relationships are correct if the **R** and **B** statistics are correct and errors are uncorrelated!

With inflation: $\mathbf{H}\mathbf{P}^b\mathbf{H}^T \rightarrow \mathbf{H}\Delta\mathbf{P}^b\mathbf{H}^T$ with $\Delta > 1$

Diagnosis of observation error statistics

Transposing, we get "observations" of Δ and σ_o^2

$$\Delta^o = \frac{(\mathbf{d}_{o-b}^T \mathbf{d}_{o-b}) - Tr(\mathbf{R})}{Tr(\mathbf{H}\mathbf{P}^b\mathbf{H}^\mathrm{T})}$$

OMB^2

$$\Delta^o = \sum_{j=1}^{p} (y_j^a - y_j^b)(y_j^o - y_j^b) / Tr(\mathbf{H}\mathbf{P}^b\mathbf{H}^T)$$

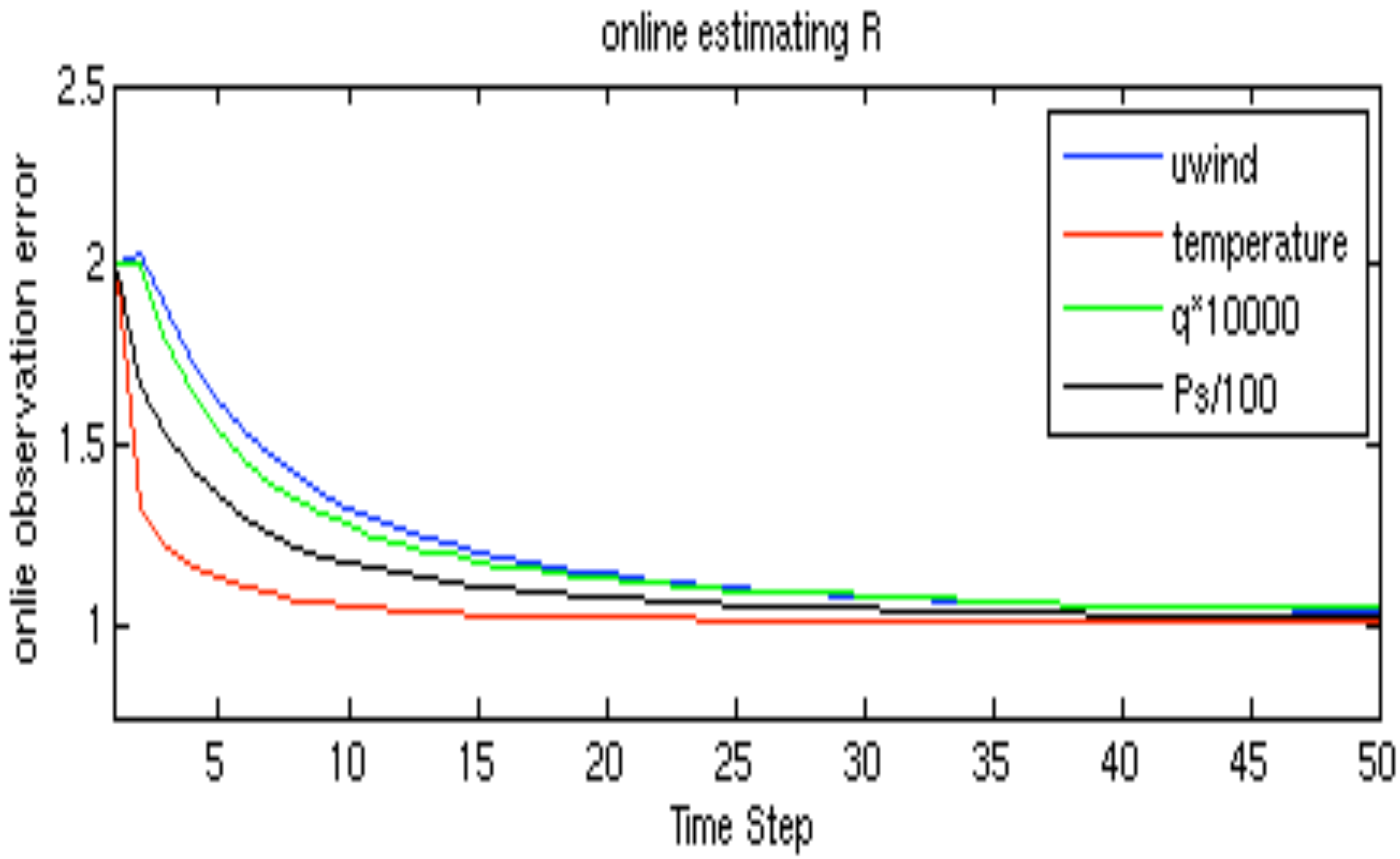
AMB*OMB

$$(\tilde{\sigma}_o)^2 = \mathbf{d}_{o-a}^T \mathbf{d}_{o-b} / p = \sum_{j=1}^{p} (y_j^o - y_j^a)(y_j^o - y_j^b) / p$$

OMA*OMB

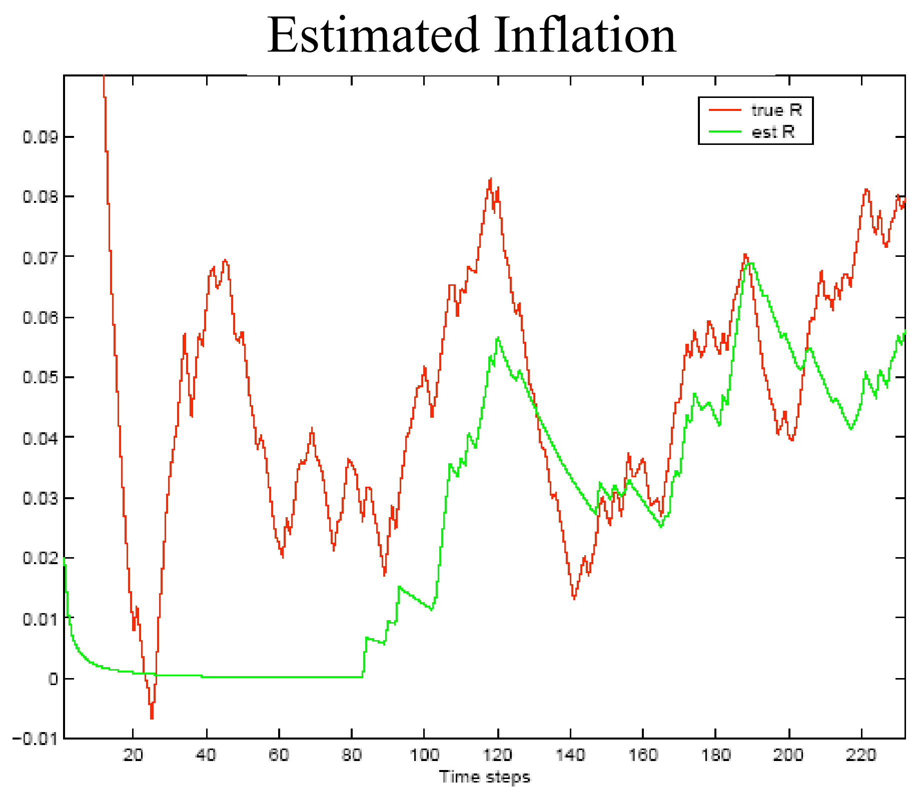
Here we use a simple KF to estimate both Δ and σ_o^2 online.

SPEEDY model: online estimated observational errors, each variable started with 2 not 1.



The original wrongly specified R converges to the correct value of R quickly (in about 5-10 days)

Estimation of the inflation



Using an initially wrong R and Δ but estimating them adaptively

Using a perfect R and estimating Δ adaptively

After **R** converges, the time dependent inflation factors are quite similar

Tests with LETKF with imperfect L40 model: added random errors to the model

Error amplitude (random)	A: true σ_o^2=1.0 (tuned) constant Δ		B: true σ_o^2=1.0 adaptive Δ		C: adaptive σ_o^2 adaptive Δ		
a	Δ	RMSE	Δ	RMSE	Δ	RMSE	σ_o^2
4	0.25	0.36	0.27	0.36	0.39	0.38	0.93
20	0.45	0.47	0.41	0.47	0.38	0.48	1.02
100	1.00	0.64	0.87	0.64	0.80	0.64	1.05

The method works quite well even with very large random errors!

Tests with LETKF with imperfect L40 model: added biases to the model

Error amplitude (bias)	A: true $\sigma_o^2=1.0$ (tuned) constant Δ		B: true $\sigma_o^2=1.0$ adaptive Δ		C: adaptive σ_o^2 adaptive Δ		
α	Δ	RMSE	Δ	RMSE	Δ	RMSE	σ_o^2
1	0.35	0.40	0.31	0.42	0.35	0.41	0.96
4	1.00	0.59	0.78	0.61	0.77	0.61	1.01
7	1.50	0.68	1.11	0.71	0.81	0.80	1.36

The method works well for low biases, but not for large biases: Model bias needs to be accounted by a separate bias correction

Summary

- EnKF and 4D-Var now give similar results in Canada and in JMA (except for model bias)(Buehner et al, Miyoshi et al)
- EnKF is better than GSI with the same resolution model and needs only 4 times more CPU (Whitaker)
- EnKF is simpler and more flexible than 4D-Var. Many new ideas to further improve it have been tested in simple models:
  - Smoothing and running in place
  - A simple outer loop to deal with nonlinearities
  - Adjoint sensitivity without adjoint model
  - Coarse resolution analysis without degradation
  - Correction of model bias combined with additive inflation gives the best results
  - Can estimate simultaneously optimal inflation and ob errors

Has the time come to test EnKF in parallel?

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  - A simple outer loop to deal with nonlinearities
  - Adjoint sensitivity without adjoint model
  - Coarse resolution analysis without degradation
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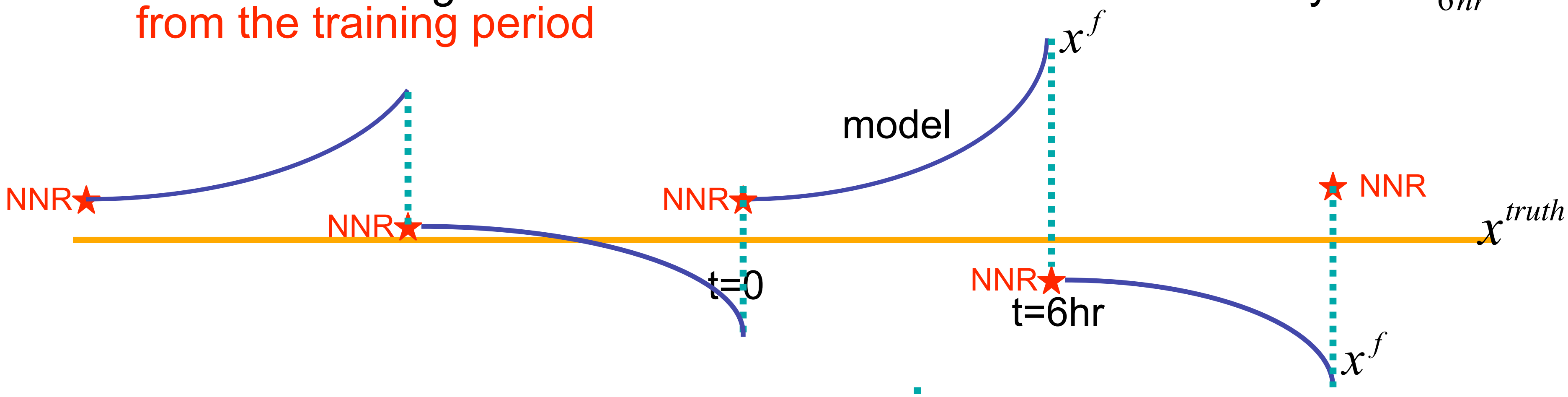
Extra Slides on Low Dim Method

Bias removal schemes (Low Dimensional Method)

2.3 Low-dim method (Danforth et al, 2007: Estimating and correcting global weather model error. *Mon. Wea. Rev, J. Atmos. Sci., 2007*)

- Generate a long time series of model forecast minus reanalysis x^e_{6hr} from the training period



We collect a large number of estimated errors and estimate from them bias, etc.

$$\varepsilon^f_{n+1} = \mathbf{x}^f_{n+1} - \mathbf{x}^t_{n+1} = \boxed{M(\mathbf{x}^a_n) - M(\mathbf{x}^t_n)} + \boxed{\mathbf{b} + \sum_{l=1}^{L} \beta_{n,l}\mathbf{e}_l + \sum_{m=1}^{M} \gamma_{n,m}\mathbf{f}_m}$$

Forecast error due to error in IC

Time-mean model bias

Diurnal model error

State dependent model error

Low-dimensional method

Include Bias, Diurnal and State-Dependent model errors:

$$\text{model error} = \mathbf{b} + \boxed{\sum_{l=1}^{2} \beta_{n,l} \mathbf{e}_l} + \sum_{m=1}^{10} \gamma_{n,m} \mathbf{f}_m$$

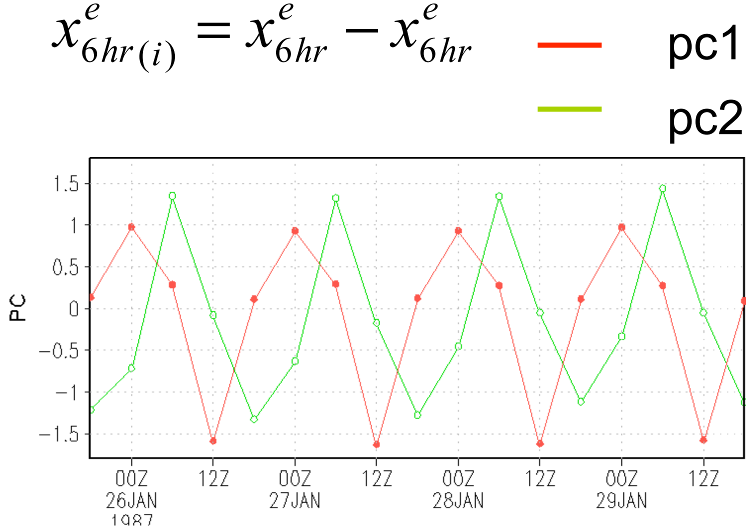
Having a large number of estimated errors allows to estimate the global model error beyond the bias

SPEEDY 6 hr model errors against NNR (diurnal cycle)

1987 Jan 1~ Feb 15

Error anomalies

$$x^{e\,'}_{6hr(i)} = x^{e}_{6hr} - \overline{x^{e}_{6hr}}$$



- For temperature at lower-levels, in addition to the time-independent bias, SPEEDY has diurnal cycle errors because it lacks diurnal radiation forcing

Leading EOFs for 925 mb TEMP